# ПAmIBIA UПIVERSITY OF SCIEMCE AMD TECHחOLOGY FACULTY OF HEALTH AND APPLIED SCIENCES 

DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics |  |
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| QUALIFICATION CODE: 07BAMS | LEVEL: 6 |
| COURSE CODE: MAP602S | COURSE NAME: <br> Mathematical Programming |
| SESSION: November 2019 | PAPER: Theory |
| DURATION: 3 hours | MARKS: 100 |


| FIRST OPPORTUNITY QUESTION PAPER |  |
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|  | MRS. S. MWEWA |
| MODERATOR: | DR. A.S EEGUNJOBI |

## INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.
2. Graph papers to be supplied by Examinations Department

## Question 1 (11 marks)

A calculator company produces handheld calculators and scientific calculators. Long-term projections indicate an expected demand of at least 150 scientific and 100 handheld calculators each day. Because of limitations on production capacity, no more than 250 scientific and 200 handheld calculators can be made daily. To satisfy a shipping contract, a minimum of 250 calculators must be shipped each day. Each scientific calculator sold results in a N\$20 loss, but each handheld calculator produces a $N \$ 50$ profit. Formulate this statement as linear programme that may yield maximum profit. (DO NOT SOLVE but variables must be unambiguously declared, and constraints must be identifiably named.)

## Question 2 (16 marks)

Consider the following linear programming model:

$$
\begin{aligned}
& \text { Maximize } P=24 a+16 b \\
& \text { Subject to } \quad 5 a+8 b \leq 40 \\
& 8 a+4 b \leq 32 \\
& a \leq 3 \\
& a ; b \geq 0
\end{aligned}
$$

2.1 Use graphical method to show that the solution of the model is $a=\frac{24}{11}, b=\frac{40}{11}, P=\frac{1216}{11}$. (It is expected that your values from the graph will be in decimal equivalence of these values.)
2.2 Hence determine the value of the slack variable for each of the three constraints. (10)

## Question 3 (9 marks)

The model in question 2 above is the dual of the primal model
Minimize $P=40 x+32 y+3 z$
Subject to $\quad 5 x+8 y+z \geq 24$
$8 x+4 y \geq 16$
$x ; y ; z \geq 0$
Use the solution of the dual to transit to and solve for the solution of this primal model. (9)

## Question 4 (10 marks)

Consider the following primal model:
Minimize $\quad z=3 x_{1}-x_{2}$
Subject to $\quad-2 x_{1}+x_{2} \leq 8$

$$
x_{1}-3 x_{2}=-2
$$

$$
x_{1}, x_{2} \geq 0
$$

Determine the dual of this primal.

## Question 5 (21 marks)

Solve the following linear programme using ONLY the two-phase method:

$$
\begin{array}{ll}
\text { Maximize } P= & 6 x+9 y \\
\text { Subject to } & 8 x+6 y \leq 48 \\
& 4 x-6 y=12 \\
& 4 x+3 y \geq 12 \\
& x ; y \geq 0
\end{array}
$$

## Question 6 (21 marks)

Consider the linear programme:

```
Maximize \(P=8 x+2 y\)
Subject to \(\quad x+2 y \leq 4 \quad\) (units of salt)
    \(2 x+y \geq 6\) (units of vitamin)
    \(x ; y \geq 0\)
```

Use graphical method (scale of $\mathbf{2 c m}$ to 1 unit on each axis) to determine
5.1 The shadow price of vitamin
5.2 The allowable increase in vitamin
5.3 The shadow price of salt
5.4 The allowable decrease in salt.

## Question 7 (12 marks)

Four jobs ( $\mathrm{J} 1, \mathrm{~J} 2, \mathrm{~J} 3$, and J 4 ) need to be executed by four workers ( $\mathrm{W} 1, \mathrm{~W} 2, \mathrm{~W} 3$, and W 4 ), one job per worker. The table below shows the cost of assigning a certain worker to a certain job.

|  | J1 | J2 | J3 | J4 |
| :--- | :---: | :---: | :---: | :---: |
| W1 | 82 | 83 | 69 | 92 |
| W2 | 77 | 37 | 49 | 92 |
| W3 | 11 | 69 | 5 | 86 |
| W4 | 8 | 9 | 98 | 23 |

Use the Hungarian method to determine which worker should be assigned what job so that cost is minimum. Also calculate the minimum cost.

